INDIAN FOREST SERVICE P (EXAM)-2014

C–HENT–N–LBSTA

MATHEMATICS

Paper—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions :

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question no. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the answer book must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

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Assume suitable data, if necessary and indicate the same clearly.

SECTION—A

Q. 1(a) Show that $u_1 = (1, -1, 0)$, $u_2 = (1, 1, 0)$ and $u_3 = (0, 1, 1)$ form a basis for $\mathbb{R}^3$. Express $(5, 3, 4)$ in terms of u_1, u_2 and u_3. 8

Q. 1(b) For the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Prove that $A^n = A^{n-2} + A^2 - I, \;\; n \geq 3$. 8

Q. 1(c) Show that the function given by

$$f(x) = \begin{cases} \dfrac{x(e^{1/x} - 1)}{(e^{1/x} + 1)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous but not differentiable at $x = 0$. 8

Q. 1(d) Evaluate $\iint\limits_R y \dfrac{\sin x}{x} \, dx \, dy$ over R where $R = \{(x, y) : y \leq x \leq \pi/2, \;\; 0 \leq y \leq \pi/2\}$. 8

Q. 1(e) Prove that the locus of a variable line which intersects the three lines :

$$y = mx,\ z = c;\quad y = -mx,\ z = -c;\quad y = z,\ mx = -c$$

is the surface $y^2 - m^2x^2 = z^2 - c^2$. **8**

Q. 2(a) Let $B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$. Find all eigen values and corresponding eigen vectors of B viewed as a matrix over :

(i) the real field R

(ii) the complex field C. 10

Q. 2(b) If $xyz = a^3$ then show that the minimum value of $x^2 + y^2 + z^2$ is $3a^2$. 10

Q. 2(c) Prove that every sphere passing through the circle $x^2 + y^2 - 2ax + r^2 = 0,\ z = 0$ cut orthogonally every sphere through the circle $x^2 + z^2 = r^2,\ y = 0$. 10

Q. 2(d) Show that the mapping $T : V_2(\bar{R}) \to V_3(\bar{R})$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation. Find the range, rank and nullity of T. 10

Q. 3(a) Examine whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Find all eigen values.

Then obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

Q. 3(b) A moving plane passes through a fixed point (2, 2, 2) and meets the coordinate axes at the points A, B, C, all away from the origin O. Find the locus of the centre of the sphere passing through the points O, A, B, C. 10

Q. 3(c) Evaluate the integral

$$I = \int_0^{\infty} 2^{-ax^2}\, dx$$

using Gamma function. 10

Q. 3(d) Prove that the equation :

$$4x^2 - y^2 + z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$$

represents a cone with vertex at (–1, –2, –3). 10

Q. 4(a) Let f be a real valued function defined on [0, 1] as follows :

$$f(x) = \begin{cases} \dfrac{1}{a^{r-1}}, & \dfrac{1}{a^r} < x \le \dfrac{1}{a^{r-1}},\ r = 1, 2, 3 \ldots\ldots \\ 0 & x = 0 \end{cases}$$

where a is an integer greater than 2. Show that $\int_0^1 f(x)\, dx$ exists and is equal to $\dfrac{a}{a+1}$. 10

Q. 4(b) Prove that the plane ax + by + cz = 0 cuts the cone yz + zx + xy = 0 in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 10

Q. 4(c) Evaluate the integral $\iint_R \frac{y}{\sqrt{x^2+y^2+1}}\,dx\,dy$ over the region R bounded between $0 \le x \le \frac{y^2}{2}$ and $0 \le y \le 2$. 10

Q. 4(d) Consider the linear mapping $F : \mathbb{R}^2 \to \mathbb{R}^2$ given as $F(x, y) = (3x + 4y, 2x - 5y)$ with usual basis.

Find the matrix associated with the linear transformation relative to the basis $S = \{u_1, u_2\}$ where $u_1 = (1, 2)$, $u_2 = (2, 3)$. 10

SECTION—B

Q. 5(a) Solve the differential equation :

$$y = 2px + p^2 y,\ p = \frac{dy}{dx}$$

and obtain the non-singular solution. 8

Q. 5(b) Solve :

$$\frac{d^4y}{dx^4} - 16y = x^4 + \sin x.$$ 8

Q. 5(c) A particle whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a' from the origin, prove that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 8

Q. 5(d) A hollow weightless hemisphere filled with liquid is suspended from a point on the rim of its base. Show that the ratio of the thrust on the plane base to the weight of the contained liquid is $12:\sqrt{73}$. 8

Q. 5(e) For three vectors show that :

$$\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})+\bar{c}\times(\bar{a}\times\bar{b})=0.$$ 8

Q. 6(a) Solve the following differential equation :

$$\frac{dy}{dx}=\frac{2y}{x}+\frac{x^3}{y}+x\tan\frac{y}{x^2}.$$ 10

Q. 6(b) An engine, working at a constant rate H, draws a load M against a resistance R. Show that the maximum speed is H/R and the time taken to attain half of this speed is $\frac{MH}{R^2}\left(\log 2-\frac{1}{2}\right)$. 10

Q. 6(c) Solve by the method of variation of parameters :

$$y'' + 3y' + 2y = x + \cos x.$$ 10

Q. 6(d) For the vector $\overline{A}=\frac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}$ examine if $\overline{A}$ is an irrotational vector. Then determine ϕ such that $\overline{A}=\nabla\phi$. 10

Q. 7(a) A solid consisting of a cone and a hemisphere on the same base rests on a rough horizontal table with the hemisphere in contact with the table. Show that the largest height of the cone so that the equilibrium is stable is $\sqrt{3}$ × radius of hemisphere. 15

Q. 7(b) Evaluate $\iint_S \nabla\times\overline{A}\cdot\overline{n}\,dS$ for $\overline{A}=(x^2+y-4)\,\hat{i}+3xy\,\hat{j}+(2xz+z^2)\,\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane. 15

Q. 7(c) Solve the D.E. :

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x.$$ 10

Q. 8(a) A semi circular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane. If the coeff. of friction is μ, prove that the greatest angle that the bounding diameter can make with the horizontal plane is :

$$\sin^{-1}\left(\frac{3\pi}{4}\,\frac{\mu+\mu^2}{1+\mu^2}\right).$$ 15

Q. 8(b) A body floating in water has volumes V_1, V_2 and V_3 above the surface when the densities of the surrounding air are ρ_1, ρ_2, ρ_3 respectively. Prove that :

$$\frac{\rho_2-\rho_3}{V_1}+\frac{\rho_3-\rho_1}{V_2}+\frac{\rho_1-\rho_2}{V_3}=0.$$ 10

Q. 8(c) Verify the divergence theorem for $\overline{A}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ over the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. 15

INDIAN FOREST SERVICE P (EXAM)-2014

C–HENT–N–LBSTB

MATHEMATICS

PAPER—II

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Section—A

1. Answer the following :

(a) If G is a group in which $(a \cdot b)^4 = a^4 \cdot b^4$, $(a \cdot b)^5 = a^5 \cdot b^5$ and $(a \cdot b)^6 = a^6 \cdot b^6$, for all $a, b \in G$, then prove that G is Abelian. 8

(b) Let f be defined on $[0, 1]$ as

$$f(x) = \begin{cases} \sqrt{1-x^2}, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$$

Find the upper and lower Riemann integrals of f over $[0, 1]$. 8

(c) Using Cauchy integral formula, evaluate

$$\int_C \frac{z+2}{(z+1)^2(z-2)}\,dz$$

where C is the circle $|z-i| = 2$. 8

(d) Obtain the initial basic feasible solution for the transportation problem by North-West corner rule : 8

Retail Shop

		R_1	R_2	R_3	R_4	R_5	*Supply*
Factory	F_1	1	9	13	36	51	50
	F_2	24	12	16	20	1	100
	F_3	14	35	1	23	26	150
		100	70	50	40	40	

(e) Find the constants a, b, c such that the function

$$f(z) = 2x^2 - 2xy - y^2 + i(ax^2 - bxy + cy^2)$$

is analytic for all $z (= x + iy)$ and express $f(z)$ in terms of z. 8

2. (a) Let J_n be the set of integers mod n. Then prove that J_n is a ring under the operations of addition and multiplication mod n. Under what conditions on n, J_n is a field? Justify your answer. 10

(b) Show that the function $f(x) = \sin\frac{1}{x}$ is continuous but not uniformly continuous on $(0, \pi)$. 15

(c) Evaluate : 15

$$\int_{|z|=1} \frac{z}{z^4 - 6z^2 + 1}\,dz$$

3. *(a)* Let R be an integral domain with unity. Prove that the units of R and $R[x]$ are same. 10

(b) Change the order of integration and evaluate $\int_{-2}^{1}\int_{y^2}^{2-y} dx\,dy$. 15

(c) Find the bilinear transformations which map the points $-1, \infty, i$ into the points—

(i) $i, 1, 1+i$

(ii) $\infty, i, 1$

(iii) $0, \infty, 1$ 15

4. *(a)* Show that the function $f(x) = \sin x$ is Riemann integrable in any interval $[0, t]$ by taking the partition $P = \left\{0, \frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, \dots, \frac{nt}{n}\right\}$ and $\int_0^t \sin x\,dx = 1 - \cos t$. 10

(b) Find the Laurent series expansion at $z = 0$ for the function

$$f(z) = \frac{1}{z^2(z^2 + 2z - 3)}$$

in the regions *(i)* $1 < |z| < 3$ and *(ii)* $|z| > 3$. 15

(c) Solve the following LPP graphically :

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 6 \\ & x_1 - x_2 \le 2 \\ & x_2 \le 4 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Write the dual problem of the above and obtain the optimal value of the objective function of the dual without actually solving it. 15

SECTION—B

5. Answer the following :

(a) Use Lagrange's formula to find the form of $f(x)$ from the following table : 8

x	0	2	3	6
$f(x)$	648	704	729	792

(b) Write a program in BASIC to integrate

$$\int_0^1 e^{-2x} \sin x\,dx$$

by Simpson's $\frac{1}{3}$rd rule with 20 subintervals. 8

(c) Show that the general solution of the pde

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

is of the form $Z(x, y) = F(x + ct) + G(x - ct)$, where F and G are arbitrary functions. 8

(d) Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega}\cdot\nabla)\vec{q} + \nu\nabla^2\vec{\Omega}$$

where $\vec{q}$ is the velocity vector with $\vec{\Omega} = \nabla \times \vec{q}$. 8

(e) Find the condition that $f(x, y, \lambda) = 0$ should be a possible system of streamlines for steady irrotational motion in two dimensions, where λ is a variable parameter. 8

6. (a) Verify that the differential equation

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

is integrable and find its primitive. 10

(b) Show that the moment of inertia of a uniform rectangular mass M and sides $2a$ and $2b$ about a diagonal is $\dfrac{2Ma^2b^2}{3(a^2 + b^2)}$. 10

(c) The values of $f(x)$ for different values of x are given as $f(1) = 4$, $f(2) = 5$, $f(7) = 5$ and $f(8) = 4$. Using Lagrange's interpolation formula, find the value of $f(6)$. Also find the value of x for which $f(x)$ is optimum. 10

(d) Write a BASIC program to sum the series $S = 1 + x + x^2 + \ldots + x^n$, for $n = 30$, 60 and 90 for the values of $x = 0\cdot1\ (0\cdot1)\ 0\cdot3$. 10

7. (a) Solve : 10

$$(D - 3D' - 2)^2 z = 2e^{2x}\cot(y + 3x)$$

(b) Solve the following system of equations : 15

$$\begin{aligned} 2x_1 + x_2 + x_3 - 2x_4 &= -10 \\ 4x_1 + 2x_3 + x_4 &= 8 \\ 3x_1 + 2x_2 + 2x_3 &= 7 \\ x_1 + 3x_2 + 2x_3 - x_4 &= -5 \end{aligned}$$

(c) A uniform rod OA of length $2a$ is free to turn about its end O, revolves with uniform angular velocity ω about a vertical axis OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or

$$\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$$ 15

8. *(a)* Using Runge-Kutta 4th order method, find y from

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0 \cdot 2,\ 0 \cdot 4$. 10

(b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M + M')g \sin \alpha}}$$

where a is the length of the plank. 15

(c) Prove that

$$\frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1$$

is a possible form for the bounding surface of a liquid and find the velocity components. 15

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